

School of Mathematical Sciences, Lahore

Problem Solving Seminar in Mathematical Analysis

February 12, 2007 at 4 p.m.

Constantin P. Niculescu: A Journey into Convex Function Theory through the Hermite-Hadamard Inequality

Abstract: The classical Hermite-Hadamard inequality gives us an estimate for the integral mean $M(f)$ of any continuous convex function $f : [a, b] \rightarrow \mathbb{R}$. Precisely, $M(f)$ lies always between the value of f at the midpoint of $[a, b]$ and the arithmetic mean of the values of f at the endpoints of $[a, b]$. This fact has a very nice generalization within Choquet's theory, by replacing the interval $[a, b]$ by any compact convex set K in a locally convex Hausdorff space, and the normalized Lebesgue measure $dx/(b-a)$ by any Borel probability measure on K . In this case, the role of the midpoint is taken by the barycenter of K . Recently, it was noticed that we can go further, by allowing the use of certain signed measures, thus extending the celebrated Steffensen-Jensen inequality. This fact, as well as the ramifications of the Hermite-Hadamard inequality, will make the subject of my talk.

C.P. Niculescu and L.-E. Persson: *Convex Functions and their Applications. A Contemporary Approach*, CMS Books in Mathematics vol. **23**, Springer-Verlag, New York, 2006.

Slides