

The American Foreign Exchange Option in Time-Dependent One-Dimensional Diffusion Model for Exchange Rate

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Abstract. The classical Garman-Kohlhagen model for the currency exchange assumes that the domestic and foreign currency risk-free interest rates are constant and the exchange rate follows a log-normal diffusion process.

In this paper we consider the general case, when exchange rate evolves according to arbitrary one-dimensional diffusion process with local volatility that is the function of time and the current exchange rate and where the domestic and foreign currency risk-free interest rates may be arbitrary continuous functions of time. First non-trivial problem we encounter in time-dependent case is the continuity in time argument of the value function of the American put option and the regularity properties of the optimal exercise boundary. We establish these properties based on systematic use of the monotonicity in volatility for the value functions of the American as well as European options with convex payoffs together with the Dynamic Programming Principle and we obtain certain type of comparison result for the value functions and corresponding exercise boundaries for the American puts with different strikes, maturities and volatilities.

Starting from the latter fact that the optimal exercise boundary curve is left continuous with right-hand limits we give a mathematically rigorous and transparent derivation of the significant early exercise premium representation for the value function of the American foreign exchange put option as the sum of the European put option value function and the early exercise premium.

The proof essentially relies on the particular property of the stochastic integral with respect to arbitrary continuous semimartingale over the predictable subsets of its zeros. We derive from the latter the nonlinear integral equation for the optimal exercise boundary which can be studied by numerical methods.