THE STRONG DUAL VARIANT OF A ROLEWICZ-BARBASHIN'S THEOREM FOR EXPONENTIALLY BOUNDED EVOLUTION FAMILIES ON BANACH SPACES

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ABSTRACT. In a recent paper ([?]) it is shown that an exponentially bounded evolution family $\mathbf{U} = \{U(t,s)\}_{t\geq s\geq 0}$ of bounded linear operators acting on a Banach space X is uniformly exponentially stable if and only if there exists $q \in [1,\infty)$ such that

$$\sup_{t\geq 0}\left(\int_0^t ||U(t,\tau)^*x^*||^q d\tau\right)^{\frac{1}{q}} = M(x^*) < \infty, \quad \forall x^* \in X^*.$$

Here we shall prove the following result. Let J be either of $\mathbf{R}$ or $\mathbf{R}_+$ and let $\Delta_J := \{(t,s) : t,s \in J \text{ and } t \geq s\}$. By $E(J)$ shall denote a rearrangement invariant Banach function space whose dual $E'(J)$ verifies the condition

$$\lim_{t\to\infty} \left\|\chi_{[0,t]}(\cdot)\right\|_{E'(J)} = \infty.$$

If $U(t,\tau) = 0$ whenever $t < \tau$ and for each $x^* \in X^*$ and each $t \geq s$ the map $\chi_{[s,t]}(\cdot)||U(t,\cdot)^*x^*||$ defines an element of the space $E'(J)$ and if in addition

$$\sup_{(t,s)\in\Delta_J} \left\|\chi_{[s,t]}(\cdot)||U(t,\cdot)^*x^*||\right\|_{E'(J)} < \infty \quad \text{for all } x^* \in X^*,$$

then the family $\{U(t,s) : t \geq s\}$ is uniformly exponentially stable. As a corollary we get a dual and non-autonomous variant of a Rolewicz's theorem. In particular we shall extend the quoted result from [?] for $0 < q < 1$. Some related results for ρ-periodic evolution families, with fixed positive ρ, are also given.

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