

DUAL INTEGRAL CHARACTERIZATIONS FOR NONUNIFORM EXPONENTIAL STABILITY OF EVOLUTION FAMILIES ON BANACH SPACES

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ABSTRACT. Let J be one of the intervals \mathbf{R} , $\mathbf{R}_+ : [0, \infty)$ or $\mathbf{R}_- = (-\infty, 0]$. We shall prove that an evolution family $\mathbf{U} = \{U(t, s)\}_{t \geq s}$ satisfying some natural assumptions is non-uniformly exponentially stable if there exists a positive real number α and a nondecreasing function $\phi : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ with $\phi(t) > 0$ for all $t > 0$ and such that

$$\sup_{t > s} \int_0^{t-s} \phi(e^{\alpha u} \|U(s+u, u)x\|) du = M_\phi(s) < \infty, \quad \forall s \in J, \forall x \in X, \|x\| \leq 1.$$

The symmetric theorem reaches the same line under the assumption that

$$\sup_{t > s} \int_s^t \phi(e^{\alpha t} e^{-\alpha \tau} \|U(t, \tau)^* x^*\|) d\tau = L_\phi(s) < \infty, \quad \forall s \in J, \forall x^* \in X^*, \|x^*\| \leq 1.$$

If ϕ is a convex function then the converse statements are obviously true.

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