

A CONDITION OF UNIFORM EXPONENTIAL STABILITY FOR SEMIGROUPS

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ABSTRACT. Let (L^Φ, ρ^Φ) be an Orlicz space over \mathbf{R}_+ , E^* its dual space and $\mathbf{T} = \{T(t)\}_{t \geq 0}$ be a strongly continuous semigroup on a complex Hilbert space H . We shall prove that if

$$\liminf_{\alpha \downarrow 0} [\alpha \|\exp_{-\alpha}\|_{E^*}] = 0$$

and

$$\sup_{\|x\|, \|y\| \leq 1} \rho^\Phi(|\langle T(\cdot)x, y \rangle|) \leq M < \infty,$$

then

$$\omega_0(\mathbf{T}) \leq M_\beta := \beta - (2M \|\exp_{-\beta}\|_{E^*})^{-1} < 0$$

for some suitable positive number β . The connection with well posed abstract Cauchy problems in Hilbert spaces is also discussed.

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