

Multi-level distance labelings for helm graphs *

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Abstract

For a graph G and any two vertices u and v in G , let $d_G(u, v)$ denote the distance between them and let $\text{diam}(G)$ be the diameter of G . A multi-level distance labeling (or radio labeling) for G is a function f that assigns to each vertex of G a positive integer such that for any two distinct vertices u and v , $d_G(u, v) + |f(u) - f(v)| \geq \text{diam}(G) + 1$. The largest positive integer in the range of f is called the span of f . The radio number of G , denoted $rn(G)$, is the minimum span of a multi-level distance labeling for G .

A helm graph H_n is obtained from the wheel W_n by attaching a vertex of degree one to each of the n vertices of the cycle of the wheel. In this paper the radio number of the helm graph is determined for every $n \geq 3$: $rn(H_3) = 13$, $rn(H_4) = 21$ and $rn(H_n) = 4n + 2$ for every $n \geq 5$. Also, a lower bound of $rn(G)$ related to the length of a maximum hamiltonian path in the graph of distances of G is proposed.

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