

Hilbert Transform: A New Integral Formula

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Introduction

It is shown in Quantum statistical mechanics [1] that the spectral function of one-particle states in general case can be represented in the form

$$A(\vec{p}, \omega) = \frac{\gamma(\vec{p}, \omega)}{(\omega - \sigma(\vec{p}, \omega))^2 + \frac{1}{4}\gamma(\vec{p}, \omega)^2},$$

where real functions γ and σ are related through Hilbert's transform [2]. The spectral function plays central role in Quantum statistics. It was proved [1] that

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} A(\vec{p}, \omega) d\omega = 1$$

for any \vec{p} .

Recently, H.S.Köhler [3] has stated a Conjecture: The sufficient condition for the validity of the above equality is that the functions γ and σ be related through the Hilbert transform.

A proof of the Conjecture is not known to exist. In the present paper we concern only pure mathematical aspects of the question. We show that in general case the Conjecture is not valid: for a certain set of functions γ and σ , the equality holds while for some others it fails.