

**CIMPA-UNESCO-ASSMS-NCM**  
**SCHOOL OF DISCRETE AND COMPUTATIONAL GEOMETRY**  
**March 23-30, 2008**  
**Abdus Salam School of Mathematical Sciences, GC University Lahore, Pakistan.**

**The abstracts of the mini-courses are**

- **Imre Bárány** (London, UK)

**COMBINATORIAL CONVEXITY**

The lectures start by introducing basic results in combinatorial convexity, for instance the Caratheodory, Helly, and Radon theorems, their connections and applications. Further developments will also be explained, including the fractional Helly theorem, and coloured versions of Caratheodory and Helly, together with their applications.

This will lead to Tverberg's theorem, another central topic in combinatorial convexity. I will also talk about the halving-set problem and its connection with a colourful version of Tverberg's theorem.

- **Andras Bezdek** (Auburn, USA)

**DISCRETE GEOMETRY VIA CYLINDERS AND SPHERES**

Most of the problems studied in discrete geometry concern the properties of arrangements of basic geometric figures like points, lines, circles, convex sets etc. One lecture will review the basics of spherical geometry, then three lectures, each using the example of cylinders will introduce different problem circles of discrete geometry:

1. The basics of spherical geometry (Gerard theorem, Lexell's theorem and analogies between spherical and planar geometry).

2. Finding reasonably good upper and lower bounds for the packing densities for the family of short cylinders, convex cones is a typical problem in discrete geometry. We explain what is known concerning cylinder packings.

3. Littlewood (1968) proposed the problem of determining the maximum number of congruent infinite cylinders that can be arranged in 3-space so that any two of them are touching. The status of this problem will be discussed, together with several variants.

4. The celebrated Tarski's plank theorem says that if the unit disc is covered by planks, then the sum of the widths of the planks is at least 2. We consider variants of this problem to show the connection of Tarski's plank problem and cylinders.

- **N. Dolbilin** (Moscow, RUSSIA)  
**POLYHEDRA AND TILINGS OF SPACE: CLASSICAL AND NEW THEOREMS**

In the theory of convex polyhedra there are four basic theorems: Euler's, Cauchy's, Minkowski's, and Alexandrov's theorems. We shall explain the meaning of these theorems and to give sketches of proofs for the first two of them. Then we present applications of some of these theorems to the theory of tilings. It is also planned to tell about recent developments of Minkowski's theorem. Besides, we shall discuss new results on polyhedra (Bleecker's and Sabitov's theorems and related results). In particular, a striking open problem will be considered

- **G. Fejes Tóth** (Budapest, HUNGARY)  
**AN INTRODUCTION TO PACKING AND COVERING**

The course will start with the classical results of Thue and Kershner about the densest packing of congruent circles and the thinnest covering by congruent circles. We continue by investigating packings and coverings by circles of not too different size. After proving Dowker's theorems we proceed to generalizations to packings and coverings by congruent copies of a convex disc. A further generalization of the theorems of Thue and Kershner, the Moment Theorem will be proved and applications of it will be discussed. If time permits, we finish with a detour to higher dimensions, proving Bichfeldt's density bound for ball packings.

- **W. Kuperberg** (Auburn, USA)  
**SOME ELEMENTARY PROBLEMS ON PACKING AND TILING**

The contents would include the problem of optimal packing of congruent balls in a cubical container in  $d$  dimensions and its relations to Hadamard matrices. This should take at least two lectures. The remaining time I would spend talking about tilings in the plane and in 3-space. In the planar case, I would mainly present the notion of  $p$ -hexagons and their applications, and in space I would show a few examples and present some open problems, including the problems about convex sets touching other convex sets, such as cylinders touching a ball, and cylinders that touch each other.

- **Janos Pach** (New York, USA)  
**EXTREMAL COMBINATORICS AND GEOMETRY. FORBIDDEN PATTERNS AND DISTANCES**

1. According to Euler's formula, every planar graph with  $n$  vertices has at most  $O(n)$  edges. How much can we relax the condition of planarity without violating the conclusion? After surveying some classical and recent results of this kind, we

prove that every graph of  $n$  vertices, which can be drawn in the plane without three pairwise crossing edges, has at most  $O(n)$  edges. What happens if the forbidden pattern consists of  $k > 3$  pairwise crossing edges? These questions can be regarded as dual counterparts of some old problems of Erdos, Kupitz, Perles, and others. Why are they interesting?

2. In 1946, in a paper published in the American Mathematical Monthly, Erdos raised the following innocent looking question: At most how many times can a given distance occur among  $n$  points in the plane? This and many similar geometric and number-theoretic questions have motivated a lot of research in extremal graph and hypergraph theory and have led to discovery of several important combinatorial tools. Nevertheless, we are still far from having a satisfactory answer to Erdos's original question. All known approaches to this problem are based on forbidden pattern arguments. We outline some old and new ideas leading to the best known bound of  $O(n^{\{4/3\}})$ , and discuss whether such arguments can ever lead to the truth.

- **T. Zamfirescu** (Dortmund, GERMANY)

### **ACUTE TRIANGULATIONS**

The discussion of acute triangulations has one of its origins in a problem of Stover reported in 1960 by Gardner in his Mathematical Games section of the Scientific American. There the question was raised whether a triangle with one obtuse angle can be cut into smaller triangles, all of them acute. In the same year, independently, Burago and Zalgaller investigated in considerable depth acute triangulations of polygonal complexes. In 1980, Cassidy and Lord considered acute triangulations of the square. More recently, Maehara and Yuan investigated acute triangulations of quadrilaterals and other polygons. On the other hand, compact convex surfaces have also been triangulated. Acute triangulations of all Platonic surfaces were investigated by Itoh and Zamfirescu. They and Hangan raised the following natural and not easy problem. Does there exist a number  $N$  such that every compact surface in 3-space admits an acute triangulation with at most  $N$  triangles? We shall present various results which are contributions towards a solution of this problem.