**How to use this program:**

1. Download Singular (if not already installed in your computer)
2. Download braid.lib
3. Start Singular
4. Load the library braid.lib (by giving the required path e.g if "braid.lib" is saved in some folder xyz inside Disk D of your computer then the path will be: `LIB"D:/xyz/braid.lib";`)
5. The input of the word $\Delta_3^r x_1^{s_1} x_2^{s_2} \cdots x_{i_h}^{s_h}$ of $\mathcal{B}_3$ ($r, s_i \in \mathbb{Z}$, $s_i \neq 0$ for $i > 1$, $s_i$ is allowed to be 0 to hint that the word starts with $x_2$) is: `list L` $= \mathtt{r}, \mathtt{s_1}, \mathtt{s_2}, \cdots, \mathtt{s_h}$ (you can choose any other name for $L$).
6. **Artin(L);** computes the Artin-invariant of the word corresponding to $L$. For example $V_1 = \Delta_3^{-6} x_1^5 x_2^{-3} x_1^2 x_2^2 x_1$ then the program works as follow:

Input:

> list `L=-6,5,-3,2,2,1;`
> `Artin(L);`

Output:

```
[1]:
  -9
[2]:
  7
[3]:
  2
[4]:
  2
[5]:
  3
[6]:
  2
>
```

The above out-put shows that $(-9; (7, 2, 2, 3, 2))$ is the Artin-invariant corresponding to $V_1$ or in other words $\Delta_3^{-9} x_1^7 x_2^2 x_1^2 x_2^3 x_1^2$ is the smallest summit word in summit set of $V_1$.

7. **nf(K);** computes the Garside normal form of the word corresponding to $K$. For example $V_2 = x_1^{-3}x_2^{-2}x_1^5$ then the program works as follow:
Input:
> `list K = 0,-3,-2,5;` (0 at the beginning of the list $K$ indicates that $r = 0$ for word $V_2$)
> `nf(K);`
Output:

```
[1]:
  -4
[2]:
  0
[3]:
  1
[4]:
  2
[5]:
  3
[6]:
  6
```

> The above out-put shows that $= \Delta_3^{-4}x_2x_1^2x_2^3x_1^6$ is the Garside normal form of $V_2$.
8.**sw(J);** computes a summit word in the summit set of the word corresponding to $J$. For example $V_3 = \Delta_3^{-3}x_2^5x_1^2x_2^{-1}$ then the program works as follow:
Input:
> `list J=-3,0,5,2,-1;` (0 as second entry of the list $J$ indicates that $x_{i_1} = x_2$ in $V_3$).
> `sw(J);`
Output:

```
[1]:
  -4
[2]:
```

```
  5
[3]:
  3
[4]:
  1
>
```

The above out-put shows that $= \Delta_3^{-4} x_1^5 x_2^3 x_1$ is a summit word in the summit set of $V_3$.

9. **sameNf($M, N$);** tells whether the words corresponding to $M$ and $N$ are equal or not. For example $V_4 = x_2^5 x_1^2 x_1^{-1}$ and $V_5 = x_1^5 x_2^3 x_1^{-2}$ then the program works as follow:

Input:

```
> list M=0,0,5,2,-1;
> list N=0,5,3,-2;
sameNf(M,N;)
```

Output:

```
no
>
```

The "no" as out-put shows that $V_4$ and $V_5$ are not equal.

9. **sameConju($X, Y$);** tells whether the words corresponding to $X$ and $Y$ are conjugate or not. For example $V_6 = x_1 x_2$ and $V_7 = x_2 x_1$ then the program works as follow:

Input:

```
> list X = 0,1,1;
> list Y = 0,0,1,1;
sameConju(X,Y;)
```

Output:

```
yes
>
```

The "yes" as out-put shows that $V_6$ and $V_7$ are conjugate.

This program is based on the results of my research paper "Conjugacy Classes of 3-Braid Group", arXiv:0801.4443v1

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For any comments please write to usman76swat@yahoo.com